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**On the Simultaneous Emergence of Money and the State**

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# On the Simultaneous Emergence of Money and the State

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## Abstract

We construct a infinite-horizon political game where the production of a public good is delegated to a politician. The politician is controlled by finitely many citizens who, on the other hand, trade commodities and pay taxes on a voluntary basis. We provide conditions in terms of heterogenous beliefs under which one single commodity is used both as a universal means of exchange and a means to pay taxes. These provide an analytical framework for the understanding of money as originating both from the private and the public sector simultaneously.

**Keywords:** Money, strategic market game, political economy, heterogenous beliefs.

**JEL Classification Numbers :** D50, E40, E50, E58.

## 1 Introduction

There are at least two competing views about the origin of money. On the one hand, part of the literature claims that governments first issued money, which would place it in the civic realm and the world of politics. This argument stems partly from the early practices of paying obligations to religious or state authorities. As ? points out, the primitive notion of *Wergeld* (worth payment) referred to a compensation for injuries and damages in communal or tribal societies. Ingham draws the following conclusion:

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All evidence points to the historical origins of money as a means of calculating obligations and debts in pre-market tribal and clan society. Early settled agricultural societies developed a more complex division of labour than the hunters and gatherers, generating a surplus that was distributed unequally. Measures for the assessment of differential social and political obligations were developed. These varied by the nature of the transgression and the status of the injured party, and formed the conceptual basis for money of account.<sup>1</sup>

According to this view, this first function of money was to pay debts to the state, and the establishment of the money of account was a means to uniformly measure the debt to be paid. this assignment of payment by the state might have easily provided a background for a second early practice: paying taxes. Only later did money function as a medium for the exchange of commodities. Furthermore, its capacity to serve as a medium of exchange was based on its credibility, which was maintained by the state.<sup>2</sup>

This view differs from the goldsmith story, the traditional textbook explanation of the origin of paper money (see, e.g., ?). According to this story, once merchants had paper receipts instead of gold, the goldsmith “decided that they could get along by keeping only a fraction of their total deposits on reserve in their vaults and lending out the balance, they acquired the ability to create money” (p. 227). Fractional reserve banking, so goes the argument, means that money is credit and originates in the (private) banking system. Who should we therefore turn to: the government or the banks ? Conversely, we could ask whether a state did really exist before it started using some device, called money, in order to compute its liabilities.

According to ?, if money is indeed credit, it seems that the government story and the goldsmith story are actually telling the same narrative —the story of credit and debt. However, the traditional understanding of the goldsmith story is that the value of the receipts or paper money is backed by gold —a commodity. If money is viewed ultimately as a commodity (a substitute for gold or any physical asset), then the goldsmith story and the government view are not quite the same. This can be best viewed in the celebrated debate between Goldbugs and Greenbacks after the US Civil War. Farmers and laborers from the western and southern states (the Greenbacks) wanted the government to issue more greenbacks for the creation of wealth, and had a credit-view on money. The eastern-based bankers (known as Goldbugs) wanted, quite to the contrary, a return to a gold-backed currency.<sup>3</sup> Here, the commodity-view of money is clearly identified with banks playing the role of the foundational institution for the origin of money; similarly, the credit-view can be traced back to the government

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<sup>1</sup>?, p. 105.

<sup>2</sup>See also ?, who points out that during the Song dynasty in China, between the tenth and the thirteenth centuries, it was the state that created paper money.

<sup>3</sup>See, e.g., ?. Similarly, the conflict between the Banking school and the Currency school in the UK at the beginning of the nineteenth century illustrates the same division of opinion.

playing this role. We can therefore rephrase our first question: is money a provider, a means of provision or a commodity ?

This paper provides a political economy model where these two questions can be tackled. Our answer consists in providing analytical conditions under which money emerges both as a universal medium of exchange on markets *and* as a means to pay taxes. Therefore, the view sustained by our analysis is that credit-money issued by the state and commodity-money issued by banks cannot be disentangled: logically, they should appear simultaneously. In other words, money should be viewed both as a commodity *and* a promise or else money should not appear. This is not to say that, historically speaking, money indeed emerged chronologically both as a medium of trade and of taxes. However, the fact that money nowadays serves both purposes in virtually all countries should not be viewed as incidental according to the standpoint defended in this paper: it means that, eventually, the historical narrative of money has converged towards its logical potentialities.

More precisely, we deal with a market economy where, to begin with, all tradable commodities are treated symmetrically. In addition to commodities, investors can trade financial securities and may pay taxes to a politician. When a sufficiently high level of it is produced, a public good improves the welfare of all the citizens. The production and allocation of the public good is delegated to the politician. The political dilemma faced by the society is to ensure that the body to which the power of collecting taxes and producing the public good is delegated does not use it for its own interests. Here, we shall see that this “fundamental dilemma” (?) is partly resolved by the control of the politicians (say, through elections). As a consequence, we show that, along an equilibrium path, on the long run, the politician on power does indeed purchase inputs with the collected taxes in order to produce some positive amount of the public good. This can be interpreted as exhibiting the emergence of the State. It then turns out that money has a value and is used by all the traders as a universal medium of exchange. We therefore conclude that the emergence of the state and that of money are concomitant.

At variance with most of the literature devoted to optimal tax systems, here, taxes are paid on a voluntary basis. Indeed, it is well known that a State becomes legitimate only once it is able to raise taxes without having to exert any apparent violence on the citizens, i.e., whenever they voluntarily pay their due taxes.<sup>4</sup> Here, we do not take as granted the existence of such a legitimate State. Rather, we seek for the conditions of its emergence. Thus, we assume that politicians may be self-interested and that citizens can vote them out of office if dissatisfied with their performance (which conforms to the Political Economy literature, see ? or ?). But, in addition, we assume that the politician in power has no way to impose the tax level. What she decides on her own is the usage she makes from tax receipts. Either she uses them to purchase commodities

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<sup>4</sup>Obviously, indeed, the police and tax administration needed to force people to pay taxes they are unwilling to settle implies a social cost. Contemporaneous Grece illustrates this issue.

for their own, private consumption ; or she purchases inputs for the production of the public good. In general, of course, the politician in power will do both. The peoples' willingness to pay taxes will then depend upon their satisfaction with the politician's policy.

## 2 Model

We first describe the basic trading mechanism involving private goods, beads and the public good. In subsection 2.2., this stage game is embedded into an infinite-horizon political game.

### 2.1 The trading mechanism

Consider a one-shot game at time  $t$ , where players act strategically, and prices form following a mechanism à la Amir-Sahi-Shubik-Yao (?).<sup>5</sup>

The economy is populated by  $H$  households. There are  $N$  commodities traded pairwise at  $\frac{1}{2}N(N-1)$  trading-posts. Each household  $h$  is characterized by an endowment  $e^h \in \mathbb{R}_+^N$ . The action,  $a^h$ , of household  $h$  consists of a  $N \times N$  matrix  $B^h = (b_j^{h,i})_{i,j=1,\dots,N}$  such that

- (i)  $b_j^{h,i} \geq 0 \forall i, j = 1, \dots, N$ ;
- (ii)  $\sum_j b_j^{h,i} \leq e_i^h$

where  $b_j^{h,i}$  is the amount of commodity  $i$  that  $h$  sends for sale on the trading-post  $(i, j)$ . The price matrix,  $P = (p_j^i)_{i,j=1,\dots,N}$  is given by:

$$p_j^i := \begin{cases} \frac{\sum_h b_j^{h,i}}{\sum_h b_i^{h,j}} & \text{if } \sum_h b_i^{h,j} > 0; \\ 0 & \text{otherwise.} \end{cases}$$

The  $N^{\text{th}}$  commodity is called “bead”, and has an intrinsic value for consumption. We do *not* assume from the beginning that beads are used as a store of value of a universal medium of exchange. We will seek for conditions under which they can be endogenously chosen, along an equilibrium path, in order to play the role of a universal medium of exchange.

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<sup>5</sup>This is one of the rare strategic market games of which we are aware and where all commodities are treated symmetrically. (Another example is ?, which can be viewed as an extension of ? to limit-orders).

There are transaction costs. Let us denote by  $c_j^i \geq 1$  a measure of the transaction cost incurred by each trader when she purchases commodity  $i$  by means of commodity  $j$ . The final holding of player  $h$  is

$$x_i^h := e_i^h - \sum_j b_j^{h,i} + \sum_j c_j^i p_j^i b_i^{h,j}. \quad (1)$$

For simplicity, for all  $i, j < N$ ,  $c_j^i = c < 1$  and  $c_i^N = c_N^i = 1$  for every  $i$ . That is: trading with beads is costless while trading two commodities distinct from beads against each other implies some cost.

$S^h$  denotes the strategy set of player  $h$ ,  $S := \prod_h S^h$  is the set of strategy profiles. Non-trivial Nash equilibria exist in the one-shot game under standard conditions (see ?).

### *Public good and taxes*

We now flesh out the previous game by adding a public good and taxes. At each time  $t$ , one of the players, called  $h_t$ , receives the ability to produce a public good,  $G$ . This good is not marketed but gives utility to households. For simplicity there is a production function  $F : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$  mapping inputs of private goods into a unique level of public good. Given a strategy profile,  $s \in S$ , and a level of public good,  $g$ , the utility of player  $h = 1, \dots, H$  is  $u^h(x^h[s], g)$ , where  $x^h[s]$  is the final outcome of  $h$  induced by  $s \in S$  according to (??).

Player  $h_t$  can buy inputs for the production of the public good. By doing so,  $h_t$  acts as a latent Treasury. In order to finance her purchases,  $h_t$  raises taxes. Payment of taxes, however, is made on a voluntary basis. No punishment is inflicted on players who refuse to pay taxes. For this purpose,  $h_t$  sells tax receipts (good  $N+1$ ) in exchange for beads. By institutional convention, no commodity  $i < N$  can be traded against commodity  $N+1$ . The trading post  $(N, N+1)$  operates at a zero transaction cost. Tax receipts do not enter the utility function of any player.

A strategy profile of player  $h \neq h_t$  is now an  $N \times N$  matrix, as in the previous section, together with the supply,  $b_{N+1}^{h,N}$ , of beads against tax receipts. The holding-in-advance constraint for beads now reads:

$$\sum_{j=1, \dots, N+1} b_j^{h,N} \leq e_N^h. \quad (2)$$

For every  $h$ , we denote by  $\Sigma^h \subset S^h \times [0, e_N^h]$  the augmented set of strategies that fulfill (??).

Player  $h_t$  strategically chooses a fraction,  $0 \leq y_i^{h_t}[\sigma] \leq x_i^h[\sigma]$ , of her final holding,  $x_i^{h_t}[\sigma]$ , in commodity  $i$  as an input for producing the public good. If  $y_i^{h_t}[\sigma] = 0$  for every  $i = 1, \dots, N$ , then  $h_t$  is entirely “selfish”: she raises taxes solely in order to finance her own, private purchases, instead of using them to produce the public good. Let

$\tau_i^{h_t} := (x_i^{h_t}[\sigma] - y_i^{h_t}[\sigma])/x_i^{h_t}[\sigma]$ . We denote by  $\Sigma^{h_t} \subset S^{h_t} \times [0, e_N^{h_t}] \times [0, 1]^N$  the strategy set of player  $h_t$ . Given  $\sigma \in \Sigma := \prod_{h=1, \dots, h_t} \Sigma^h$ , the quantity of public good produced is

$$g[\sigma] := F\left(\left((1 - \tau_i^{h_t})x_i^{h_t}[\sigma]\right)_{i=1, \dots, N}\right).$$

And the final payoff of player  $h_t$  is  $u^{h_t}(x^{h_t}[\sigma], g[\sigma])$ .

## 2.2 The infinite-horizon game

### *Uncertainty and preferences*

We embed the previous trading mechanism in an infinite-horizon economy in discrete time, populated by finitely many types of long-lived households,  $h = 1, \dots, H$ , each of them being represented by a continuum of measure 1 of identical citizens.

In each period,  $t$ , the state of nature in the next period,  $t + 1$ , is chosen using a Markov transition matrix,  $\omega$ , with a finite set of possible states of nature  $\mathcal{S} = \{1, \dots, S\}$ . A node  $\sigma \in \mathbf{D}$  can be interpreted as a date-event pair  $(s_{t-1}, s)$ , where  $t \geq 1$  is the minimal length of a walk between  $\sigma$  and the root,  $\sigma_0$ , of the event tree. The history  $s_{t-1} \in \prod_{t'=1}^{t-1} \mathcal{S}$  is the sequence of realizations of the state of nature up to  $t - 1$ .<sup>6</sup> The  $N + 1$  commodities and the public good are perishable.<sup>8</sup> Each households' random endowment,  $e_\sigma^h \in \mathbb{R}_+^{N+1}$ , verifies  $e_{N+1, \sigma}^h = 0$  and  $e_\sigma^h > 0$  for every  $\sigma$ .<sup>9</sup>

Individual preferences at time  $t = 0$  are given by:

$$U^h(x, g) = \mathbb{E}_0^h \sum_{t=0}^{\infty} \beta^t u^h(x_t^h, g_t), \quad (3)$$

where  $x \in \ell_+^\infty(\mathbb{R}^{N \times S})$  is a uniformly bounded stream of commodity bundles,  $g \in \ell_+^\infty(\mathbb{R}^S)$ , a bounded sequence of public goods.

ASSUMPTION (C). For every  $h$ ,  $u^h(\cdot) : \mathbb{R}_+^N \rightarrow \mathbb{R}$  is  $\mathcal{C}^2$ , differentiably strictly increasing and concave, and verifies the “ruin aversion” hypothesis: For some  $\underline{\alpha}^h, \bar{g} \in \mathbb{R} \times \mathbb{R}_+$ ,

<sup>6</sup>Time, uncertainty and the revelation of information are described by an event tree, i.e., a directed graph  $(\mathbf{D}, \mathcal{A})$  consisting of a set  $\mathbf{D}$  of nodes and a set  $\mathcal{A} \subset \mathbf{D} \times \mathbf{D}$  of (oriented) arcs. Let  $\tau(\sigma)$  be the time at which node  $\sigma$  is reached, i.e.  $\tau : \mathbf{D} \rightarrow \mathbb{N}$  such that  $\sigma = (s_{t-1}, s) \mapsto t$ . Define a partial order  $\geq$  on  $\mathbf{D}$  by  $\sigma = (s_{t-1}, s) \geq \sigma' = (s_{t'-1}, s')$  if, and only if, there is a walk from  $\sigma'$  to  $\sigma$ . Of course, if  $\sigma \neq \sigma'$  and  $\sigma \geq \sigma'$ , then  $\sigma > \sigma'$ . The unique predecessor of  $\sigma$  is denoted by  $\sigma^- = (s_{t-2}, s')$ .<sup>7</sup> The set of immediate successors of  $\sigma$ , denoted by  $\sigma^+$ , is the set of nodes that are adjacent from  $\sigma$ . For any node  $\sigma \in \mathbf{D}$ , the set of all nodes with  $\sigma' \geq (>) \sigma$  is denoted by  $\mathbf{D}(\sigma)$  ( $\mathbf{D}(\sigma)^+$ ) and is itself a tree with root  $\sigma$ .

<sup>8</sup>We could allow for durable goods without impairing our results, cf. Remark ?? *infra*.

<sup>9</sup>Which means,  $e_{i, \sigma}^h \geq 0$  for every commodity  $i = 1, \dots, N$  and  $e_\sigma^h \neq 0$ .



$$\lim_{x \rightarrow \partial \mathbb{R}_+^N} u^h(x, 0) = -\infty, \text{ and } \lim_{x \rightarrow \partial \mathbb{R}_+^N} u^h(x, g) \geq \underline{\alpha}^h \quad \forall g > \bar{g}.$$

This “ruin-aversion” can be interpreted as follows: in the Middle-Age, random shocks may stand for (violent) aggressions or invasions, whose outcome was sometimes a complete loss of the households’ ownerships. In this context, the public good,  $G$ , may stand for a security provision (army, etc.). In case of a “bad state”,  $s$  (an invasion, say), somewhat could be saved from the peasants’ culture provided the king could offer some security by protecting them. In modern times, a similar interpretation holds where the security provision stands for, say, Social Security (cf. ?).

Important, also, in (??) is the superscript,  $h$ , of the expectation operator,  $\mathbb{E}^h(\cdot)$ . It means that household  $h$  evaluates her expected discounted utility with respect to her own subjective probability measure. Equation (??) can thus be rewritten:

$$\sum_{t, s^t} P^h(s^t) \beta^t u^h(x_t^h, g_t),$$

where  $P^h(s^t)$  is the probability of the history,  $s^t$ , according to  $h$ ’s beliefs.

Each citizen  $h$  can potentially become a politician.<sup>10</sup> The politician in power raises taxes in beads and decides the production of the public good.

#### *The political game*

At each time  $t$ , the timing is as follows. The economy starts with a politician in power,  $h_t$ . Then,

1. Citizens trade according to the strategic market game described in the section ???. In particular, they decide the amount of beads they provide to  $h_t$ .
2. The politician in power,  $h_t$ , chooses  $\tau^{h_t} := (\tau_i^{h_t})_{i=1, \dots, N}$  (i.e., decides the amount of public good she produces out of her purchases).
3. Elections are held and citizens jointly decide whether to keep the politician or to replace her with a new one. We abstract from the details of the institutional arrangement driving the election process. For simplicity, let us assume that, if a proportion  $\kappa \in (0, H]$  of citizens are willing to replace  $h_t$ , then  $h_t$  becomes out of office ( $\kappa = H/2$  corresponds to the simple majority rule).<sup>11</sup> In this case, another citizen  $h_{t+1}$  is chosen at random within  $T \setminus \{h_t\}$

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<sup>10</sup>This contrasts with a large body of the literature devoted to Political Economy (e.g. ?) where politicians constitute a group distinct from the representative consumer. In our paper, households are heterogenous in endowments, preferences and beliefs, and all of them can become politicians.

<sup>11</sup>Thus, at variance with (?), we allow for conflict of interest among citizens over the replacement decision.

with a uniform distribution. In a pre-democratic context, the replacement of a politician in power can be interpreted as the outcome of various processes, different from a voting procedure (such as a revolution, a putsch, or even the murder of the King...).

### 3 When money and the State emerge

At node  $\sigma = (s^t, s)$ , let

$$h^t = (s^t, (p_\tau)_{\tau < \sigma})$$

denote the public history of the game up to date  $t$ , and let  $H^t$  be the set of all such histories. A *subgame perfect equilibrium* (SPE) is given by trading actions,  $(a_h^*)_h$  at time  $t$ , given history  $h^{t-1}$ , policy decision,  $y^{h^t}$ , by the politician in power given  $h^{t-1}$ , and electoral decisions by the citizens, given history  $h^{t-1}$ , that are best responses to each other for all histories. Since we allow households to have heterogenous beliefs, the ex ante maximization of one's expected utility need not coincide with the ex post maximization of her realized utility. Thus, we focus on SPE that are renegotiation-proof (cf. ?). A SPE is *renegotiation-proof* if after any history,  $h^t$ , there does not exist another SPE that can make all payers weakly better off and some strictly better off.<sup>12</sup>

As already said, each household  $h$  has her own estimate,  $\omega^h$ , of the transition matrix. Following ?, we assume that each citizen's estimate is not too far from the truth, :

ASSUMPTION (P). For every  $h$ ,

- (i) there exist at least two states  $s, s'$  with  $\omega^h(s, s') \neq \omega(s, s')$ .
- (ii)  $\omega(s, s') = 0 \Rightarrow \omega^h(s, s') = 0 \forall s, s'$
- (iii)  $\exists \rho_h, r_h > 0$  such that  $r_h < \frac{\omega^h(s, s')}{\omega(s, s')} < \rho_- h \quad \forall s, s'$ .<sup>13</sup>

For the next result we also make the following assumption, where  $\bar{g}$  has been defined in Assumption (C) above:

ASSUMPTION (G)

$$\forall h, \exists s \mid F(e_s^h) < \bar{g}. \tag{4}$$

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<sup>12</sup>At variance with ?, however, we do not restrict ourselves to “best SPE”.

<sup>13</sup>With the convention  $0/0 := 1$ .

In words, no individual player can produce a level of public good higher than the threshold  $\bar{g}$  in every state, only by using her own initial endowment.

**Theorem 1** *Under (C), (P) and (G), along any equilibrium path induced by a non-autarkic renegotiation-proof SPE, there exists  $t$  such that no replacement of the politician in power,  $h_t$ , occurs after  $t$ , and, at each subsequent date,  $t' > t$ ,  $h_t$  produces  $g_{t'} \geq \bar{g}$ .*

**Proof.**

As a benchmark, we shall consider the perfectly competitive equilibria under complete financial markets associated with the underlying infinite-horizon economy (no public good, and no political game). Consider the situation where, in each node  $\sigma$ , there are  $S$  financial securities. Financial security  $s$  delivers one unit of good  $N$  (beads) if state  $s$  happens at time  $t + 1$  and zero units otherwise. Consumers can borrow and lend freely by buying and selling Arrow-Debreu state contingent securities, only subject to the no-Ponzi condition. A competitive equilibrium is then defined in the usual way.

On the other hand, we shall need to consider an auxiliary economy where transaction costs are incorporated into the utility functions of traders. For this purpose, let  $\tau^h \in \mathbb{R}^N$  be a net trade vector (with positive components representing purchases and negative components representing sales). Define, for every  $\ell < N$

$$\tau_\ell^h[c] := \min\{\tau_\ell^h, \frac{\tau_\ell^h}{c}\}.$$

One has:  $\tau_\ell^h(c) = \tau_\ell^h$  if  $\tau_\ell^h < 0$  and  $\tau_\ell^h(c) = \tau_\ell^h/c$  otherwise. The economy  $\mathcal{E}[c]$  with “ $c$ -diminished” trades is defined in entirely the same way as  $\mathcal{E}$ , except that each household’s utility is replaced by<sup>14</sup>

$$\forall \sigma, \quad u_{\sigma,c}^h(x) := u^h(e_\sigma^h + (x^h - e_\sigma^h)[c]).$$

**Lemma 3.1** *Every non-autarkic renegotiation-proof SPE involves perfectly competitive trades in the auxiliary economy  $\mathcal{E}[c]$ .*

**Proof.**

Since there is a continuum of each type of players, we know from ? and ?, that everything goes as if, along the equilibrium path, players would not condition their strategies over the actions of their opponents. Hence, we can focus on the conditionality over (random, past) states and public prices. Next, the transaction costs and the fact that each player is negligible imply that there is no wash sales along any equilibrium path. That is, for each player  $h$  and each pair of commodities  $(i, j)$ ,  $b_j^{h,i} b_i^{h,j} = 0$ . Indeed, suppose there is such a wash sale, then a player could obtain the same final outcome

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<sup>14</sup>Such an economy has been considered by ?.

with no wash sale at a lower cost. Since she is negligible, the change of action does not affect prices, so that the equivalent action with no wash sale is indeed profitable.

Next, the “ruin aversion” property together with the no-wash-sale property imply that the hold-in-advance constraint is never binding: no player bids her entire endowment in any commodity, i.e., .

$$\sum_j b_j^{h,i} < e_i^h. \quad (5)$$

Therefore, prices that form at each node,  $\sigma$ , are consistent, in the sense that, for any triple  $(i, j, k)$ , one has:  $p_k^i = p_j^i p_k^j$ . Indeed, suppose, on the contrary, that there exists a triple  $(i, j, k)$  with, say,  $p_k^i < p_j^i p_k^j$ . Then, player  $h$  could take advantage of this arbitrage opportunity by an obvious infinitesimal change of her action. This change is possible thanks to (??) and will not affect prices, hence will not alter the inequality  $p_k^i < p_j^i p_k^j$ .<sup>15</sup> Moreover, transaction costs do not prevent the infinitesimal deviation from being profitable, which contradicts the equilibrium character of the original strategy profile.

Since each player is negligible, this implies that the outcome of a SPE is competitive. This follows from an adaptation of ?.

□

### End of proof of the Theorem.

One can show (using the arguments of ?) that, along a perfectly competitive equilibrium (with no public good), for every player who has incorrect beliefs, almost surely, her consumption in some commodity  $i$  will go to zero as time goes to infinity. Indeed, since she makes mistakes in her expectations about future random shocks, she takes risky positions on the financial markets (despite the fact that they are complete). Sooner or later, she will bet on the incorrect state, and loose her wealth. (Cf. Proposition 1 in ?) More precisely, one shows that

$$\lim_{t \rightarrow \infty} \frac{P^h(s^t)}{P(s^t)} = 0.$$

From this, using the first-order conditions of the maximization programme of a citizen (who takes prices as given) yields, for every  $h$  and every commodity  $i$ :

$$\lim_{t \rightarrow \infty} \frac{\partial u_c^h(x^h(s^t))}{\partial x^\ell} = +\infty$$

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<sup>15</sup>This would not be true if there was a finite number of player. ? provides a counter-example. This would also fail if there were wash sales at equilibrium since a player could bid her entire initial endowment in  $i$  and repurchase part of it through her supply of another commodity in exchange for  $i$ . Notice also that a consequence of this non-arbitrage property is that the bid-ask spread is 0 on each trading-post at equilibrium.

which implies that  $x^h(s^t) \rightarrow \partial \mathbb{R}_+^N$ . If, in addition, the quantity of public good produced after a finite time,  $t$ , does not exceed  $\bar{g}$ , this cannot be a renegotiation-proof SPE because of the “ruin aversion” property: the player whose consumption comes close to the boundary,  $\partial \mathbb{R}_+^N$ , would rather not trade and keep her initial endowment. Every such player will therefore elect (in the next round) a politician who produces a quantity  $g > \bar{g}$  of public good.

For this purpose, however, we have to make sure that, along a renegotiation-proof SPE, players will indeed pay taxes. If no player pays any taxes when wealthy, then we know from (??) that, at best, an insufficient amount  $g < \bar{g}$  of public good will be produced in each period. Moreover, after a sufficiently long time, almost surely, a single player will remain (the one who, so far, never made mistakes) and all the others will remain trapped at a consumption bundle arbitrarily close to the boundary,  $\partial \mathbb{R}_+^N$ . This cannot be a renegotiation-proof SPE for the losers. Therefore, they should have rather paid some taxes when they were still wealthy in order to have some chance to get the public good later (provided they succeed in electing a benevolent politician).<sup>16</sup>

Thus, sooner or later, a majority of the players will pay taxes and remove every politician who does not produce more than  $\bar{g}$  units of public good. Now, why should any politician accept to behave in a benevolent way ? Suppose that every politician on power simply consumes all her purchases, and never produces the public good. Then, we know from the preceding considerations that each such politician will loose power after a certain time. After having been removed, a politician becomes again an anonymous household (with incorrect beliefs). Sooner or later, she will make mistakes and end up with a consumption bundle arbitrarily close to 0 (in at least one commodity) with probability 1. At this time (say  $\tau$ ), she would happily receive some public good, but, by assumption, the current politician on power at time  $\tau$  will also refuse to produce a quantity  $g > \bar{g}$  public good. Thus, our former politician should have rather accepted to produce this public good when she was on power, in order to keep being reelected and to ensure that the public good will be produced.

Finally, since after a certain time  $t^*$ , a quantity  $g > \bar{g}$  of public good is produced, this implies that, from  $t^*$  on, a positive amount of taxes is paid by citizens. Hence, beads are used in each period in order to finance taxes. Our hypothesis on the transaction costs then implies that the transaction tree will be star-shaped from  $t^*$  on.

□

That is to say : people are willing to pay taxes (hence to accept money as a universal medium of exchange) because they need a public good as a protection, not against uncertainty (markets are complete), but against their own misbeliefs. A household accepts to play the role of the government because it is safer to do it rather than incurring the risk that no public good is produced at all.

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<sup>16</sup>Along this argument, we make use of the fact that we focus on *renegotiation-proof* equilibria. Indeed, ex ante, it might be the case that no player anticipates her ruin.

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